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## A Macroeconomic Model of the Term Structure of Interest Rates in Mexico\*

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### Abstract

This paper investigates how different macroeconomic shocks affect the term-structure of interest rates in Mexico. In particular, we develop a model that combines a no-arbitrage specification of the term structure with a macroeconomic model of a small open economy. We find that shocks that are perceived to have a persistent effect on inflation affect the level of the yield curve. The effect on medium and long-term yields results from the increase in expected future short rates and in risk premia. With respect to demand shocks, our results show that a positive shock leads to an upward flattening shift in the yield curve. The flattening of the curve is explained by both the monetary policy response and the time-varying term premia.

**Keywords:** Term-Structure, No-Arbitrage, Macroeconomic Shocks.

**JEL Classification:** C13, E43, G12

### Resumen

En este artículo se investiga cómo afectan distintos choques macroeconómicos a la estructura temporal de tasas de interés en México. En particular, se desarrolla un modelo que combina una especificación de no-arbitraje de la estructura temporal de tasas con un modelo macroeconómico para una economía pequeña y abierta. Se encuentra que aquellos choques que tienen un efecto persistente sobre la inflación afectan el nivel de la curva de rendimientos. El efecto en los rendimientos de mediano y largo plazo es provocado por el incremento en las expectativas de tasas de interés futuras de corto plazo y por las primas de riesgo. Con respecto a los choques de demanda, se encuentra que un choque positivo provoca un incremento y un aplanamiento en la curva de rendimientos. El aplanamiento es explicado por la respuesta de la autoridad monetaria y por las primas de riesgo variables.

**Keywords:** Estructura-Temporal, No-Arbitraje, Choques Macroeconómicos.

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# 1 Introduction

This paper investigates how different macroeconomic shocks affect the term-structure of interest rates in Mexico. In particular, we develop and estimate a model that combines an affine no-arbitrage finance specification of the term structure in the tradition of Ang and Piazzesi (2003) with a small scale macroeconomic model for a small open economy. The affine no-arbitrage specification allows risk premia to be time-varying, while the macro model introduces structure on the dynamics of the macro variables and thus allows us to identify how structural shocks affect the economy (the no-arbitrage literature typically uses vector autoregressive processes to describe the dynamics of the state variables).

Describing the joint behavior of the yield curve and macroeconomic variables is important for bond pricing, investment decisions, fiscal and monetary policy, among others. Recent theoretical and empirical research in finance has led to a better understanding of the dynamic properties of the term structure of interest rates. Most term structure models use latent factors to explain term structure fluctuations, for example, Duffie and Kan (1996), Dai and Singleton (2000) and Duffee (2002). These models are developed under the assumption of no-arbitrage, and they can capture some important features of the yield curve by using the latent factors. However, they fail to explain what macroeconomic variables affect these latent variables. In a different approach, many empirical studies use Vector Autoregressive (VAR) models to explain the joint behavior of the term structure of interest rates and macroeconomic variables. For example, Campbell and Ammer (1993) use a VAR model to study excess stock and bond returns, and their results show that stock and bond returns in the US are driven largely by news about future excess stock returns and inflation. Evans and Marshall (2001) also use a VAR model to investigate the impacts of monetary and real shocks on various interest rates. They find that the shocks to monetary policy have a pronounced but transitory effect on short-term interest rates, with almost no effect on long-term interest rates. In contrast, the shocks to employment have a long-lived impact on interest rates across the maturity spectrum. VAR models are useful to examine the impact of macroeconomic shocks on various interest rates through impulse response functions. However, there are several disadvantages to using VAR models to study the term structure of interest rates. First, one

can only study the effects of macroeconomic variables on those yields of maturities that are included in the model. The VAR models do not describe how yields of maturities not included will respond to changes in the macroeconomic variables. Second, the predicted movements of the yields with different maturities in the VAR models may not rule out arbitrage, since the unrestricted VAR models do not require that the movement of various interest rates provide no-arbitrage opportunities. By contrast, an arbitrage free term structure model provides a complete description of how the yields of all maturities respond to the shocks to the underlying state variables.

In this paper, we combine an affine no-arbitrage finance specification of the term structure with a structural macroeconomic model for a small open economy. We incorporate macroeconomic variables as factors in a term structure model by using a factor representation for the pricing kernel, which prices all bonds in the economy. This is a direct and tractable way to modelling how macro factors affect bond prices.

Our article is part of a rapidly growing literature exploring the relation between the term structure and macroeconomic dynamics. Kozicki and Tinsley (2001) and Ang and Piazzesi (2003) were among the first to incorporate macroeconomic factors in a term structure model. Our paper differs from these articles in that all the macro variables obey a set of structural macroeconomic relations. This facilitates a meaningful economic interpretation of the term structure dynamics. For instance, we can trace the impact of macroeconomic shocks on the term structure of interest rates. Moreover, the implied interactions between macroeconomic variables and the term structure of interest rates are more general in our framework than in the articles we mentioned.

Three related studies are Rudebusch and Wu (2004), Hordahl, Tristani and Vestin (2006), and Bekaert, Cho and Moreno (2005), who also append a term structure model to a New-Keynesian macro model. All these papers study the joint dynamics of bond yields and macroeconomic variables in a closed economy framework.

In this paper, we investigate the joint dynamics of bond yields and macroeconomic variables in a small open economy framework. The domestic yield curve is modeled in the affine term structural framework, and the price of risk depends on both domestic and foreign macroeconomic variables.

Our main findings are as follows. As in developed markets (Ang and Piazzesi 2003), results from the estimation of the model show that term premia are countercyclical, and that they increase with the level of the inflation rate. In addition, our model delivers strong contemporaneous responses of the entire term structure to various macroeconomic shocks. For example, shocks that are perceived to have a persistent effect on inflation (i.e. a persistent cost-push shock) affect the level of the yield curve. The effect on medium and long-term yields results from the increase in expected future short rates and in risk premia. With respect to demand shocks, our results show that a positive demand shock leads to an upward flattening shift in the yield curve. In this case, the flattening of the curve is explained by both the monetary policy response (the monetary authority increases the short-term interest rate following this shock), and the time-varying term premia.

The remainder of the paper is organized as follows. Sections 2 and 3 outline the structural macroeconomic model and the term structure model respectively. Section 4 discusses the estimation methodology, while section 5 presents and analyzes the results. Section 6 concludes.

## 2 Macroeconomic Model

We present a small open economy New-Keynesian model featuring a Phillips curve, an IS curve and a monetary policy rule with two additions. First, we assume that total inflation is a weighted average of core and non-core inflation. The dynamics of core inflation are described by a New-Keynesian Phillips curve, while non-core inflation follows an AR(1) process. Second, given the empirical evidence against the uncovered interest rate parity (UIRP), we incorporate the lagged real exchange rate in the UIRP equation.

### 2.1 Aggregate Supply

The aggregate supply equation describes the dynamics of inflation. The aggregate supply equation that we use in the model is of the Phillips curve type estimated by Svensson (1998). We can derive a forward looking Phillips curve linking inflation to future expected inflation and the output gap using Calvo's pricing framework with monopolistic competition in the

intermediate goods markets. If we assume that the fraction of price-setters which does not adjust prices optimally, indexes their prices to past inflation, we obtain endogenous persistence in the AS equation. Consequently, we obtain a standard New-Keynesian aggregate supply curve relating core inflation to the output gap:

$$\pi_t^c = a_1\pi_{t-1}^c + a_2E_t(\pi_{t+1}^c) + a_3x_t + \epsilon_t^{AS} \quad (1)$$

where  $\pi_t^c$  is core inflation,  $x_t$  is the output gap, and  $\epsilon_t^{AS}$  is an exogenous supply shock.  $a_3$  captures the short-run tradeoff between inflation and the output gap and  $a_1$  characterizes the endogenous persistence of inflation, where  $a_1 + a_2 = 1$  since the AS curve satisfies the property of dynamic homogeneity.

Since we are modelling a SOE, we need to incorporate the effects of the exchange rate on inflation. Several authors like McCallum and Nelson (2001), and Galí and Monacelli (2005) have developed SOE economy versions of the AS equation:

$$\pi_t^c = a_1\pi_{t-1}^c + a_2E_t(\pi_{t+1}^c) + a_3x_t + a_4(\Delta e_t + \pi_t^{USA}) + \epsilon_t^{AS} \quad (2)$$

where  $\Delta e_t$  denotes the change in the nominal exchange rate,  $\pi_t^{USA}$  denotes U.S. inflation, and the parameter  $a_4$  represents the pass-through of the nominal exchange rate and U.S. inflation to domestic inflation. Since the AS curve satisfies the property of dynamic homogeneity  $a_1 + a_2 + a_4 = 1$ .

The change in the real exchange rate is defined as follows:

$$\Delta q_t = \Delta e_t + \pi_t^{USA} - \pi_t \quad (3)$$

where  $q_t$  denotes the real exchange rate, a higher  $q_t$  denotes a depreciation of the SOE currency.  $\pi_t$  denotes total inflation, and is equal to:

$$\pi_t = \omega\pi_t^c + (1 - \omega)\pi_t^{nc} \quad (4)$$

We assume that non-core inflation follows an AR(1) process:

$$\pi_{t+1}^{nc} = \delta_0 + \delta_1 \pi_t^{nc} + \epsilon_{t+1}^{nc} \quad (5)$$

## 2.2 Aggregate Demand

In a closed economy, the aggregate demand equation is usually derived from the first order conditions for a representative agent in a general equilibrium model. Since standard approaches fail to match the persistence in the output gap, recent studies like Fuhrer (2000), and Cho and Moreno (2005) derive an alternative IS equation from a utility maximizing framework with external habit formation:

$$x_t = b_1 x_{t-1} + b_2 E_t(x_{t+1}) + b_3 (i_t - E_t \pi_{t+1}) + \epsilon_t^{IS} \quad (6)$$

where  $i_t$  is the short-term interest rate. The residual  $\epsilon_t^{IS}$  is an aggregate demand shock, in this equation the habit formation specification imparts endogenous persistence to the output gap. The parameters  $b_1$  and  $b_2$  depend on the level of habit persistence and the risk aversion parameter.

We follow McCallum and Nelson (2001), and Gali and Monacelli (2005) and specify the aggregate demand dynamics as:

$$x_t = b_1 x_{t-1} + b_2 E_t(x_{t+1}) + b_3 (i_t - E_t \pi_{t+1}) + b_4 x_t^{USA} + b_5 q_t + \epsilon_t^{IS} \quad (7)$$

The IS equation provides a description of the dynamics of aggregate demand, which is affected by movements in the short-term real interest rate, the real exchange rate and the U.S. output gap. The forward looking term captures the intertemporal smoothing motives characterizing consumption.

## 2.3 Monetary Policy Rule

We assume that the monetary authority sets the short-term interest rate according to a simple Taylor rule:

$$i_t = \rho i_{t-1} + (1 - \rho) [\bar{i}_t + d_1 (\pi_t - \pi_t^*) + d_2 x_t] + \epsilon_t^{MP} \quad (8)$$

The central bank reacts to high inflation and to deviations of output from its trend. The parameter  $d_1$  measures the response of the Central bank to inflation, while  $d_2$  describes its reaction to output gap fluctuations.  $\pi_t^*$  is a time-varying inflation target and  $\bar{i}_t$  is the desired level of the nominal interest rate that would prevail when  $\pi_t = \pi_t^*$  and  $x_t = 0$ . We assume that  $\pi_t^*$  and  $\bar{i}_t$  are constant. The parameter  $\rho$  captures the tendency by central banks to smooth interest rate changes (see Clarida, Gali and Gertler (1999)), and  $\epsilon_t^{MP}$  is an exogenous monetary policy shock.

## 2.4 Real Exchange Rate

Uncovered interest parity predicts that high yield currencies should be expected to depreciate. It also predicts that, ceteris paribus, a real interest rate increase should appreciate the RER. Nevertheless, there appears to be overwhelming empirical evidence against the UIRP. Given the empirical evidence against UIRP we incorporate the lagged real exchange rate in the UIRP equation:

$$q_t = c_1 q_{t-1} + c_2 [E_t(q_{t+1}) + (i_t^{USA} - E_t \pi_{t+1}^{USA}) - (i_t - E_t \pi_{t+1})] + \epsilon_t^q \quad (9)$$

if  $c_1 = 0$ , and  $c_2 = 1$ , then UIRP holds.  $\epsilon_t^q$  is an exogenous real exchange rate shock.

## 2.5 Exogenous Variables

We assume that the U.S. variables  $\pi_t^{USA}, x_t^{USA}, i_t^{USA}$  are exogenous and follow a VAR(2) process. Both domestic and foreign structural shocks are assumed to be independent and identically distributed with homoskedastic variances. Our macroeconomic model can be



expressed in matrix form as:

$$Q \begin{bmatrix} X_{1,t+1} \\ E_t X_{2,t+1} \end{bmatrix} = Z \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} + B i_t + \begin{bmatrix} \xi_{1,t+1} \\ 0 \end{bmatrix} \quad (10)$$

where  $X_{1,t}$  is a vector of predetermined variables,  $X_{2,t}$  is a vector of forward-looking variables,  $i_t$  is the policy instrument and  $\xi_{1,t+1}$  is a vector of independent and identically distributed shocks. We also assume that  $\xi_{1,t} \rightsquigarrow N(0, \Sigma)$ , where  $\Sigma$  is a diagonal matrix with constant variances. The short-term nominal interest rate can be written in the feedback form:

$$i_t = -F \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} \quad (11)$$

The coefficients of matrices  $Q$ ,  $Z$ ,  $B$  and  $F$  are defined by the structural equations of the domestic and foreign country macroeconomic variables. Under regularity conditions, the solution of the model can be obtained numerically following standard methods. The rational expectations equilibrium can be written as a first-order VAR:

$$X_t = c + \Omega X_{t-1} + \Gamma \xi_t \quad (12)$$

where  $X_t = (\pi_t^c, x_t, i_t, q_t, \pi_t^{nc}, \pi_t^{USA}, x_t^{USA}, i_t^{USA})'$  and  $\xi_t = (\epsilon_t^{AS}, \epsilon_t^{IS}, \epsilon_t^{MP}, \epsilon_t^q, \epsilon_t^{\pi^{nc}}, \epsilon_t^{\pi^{USA}}, \epsilon_t^{x^{USA}}, \epsilon_t^{i^{USA}})'$ .

Hence, the implied model dynamics are a VAR subject to a set of non-linear restrictions. Note that  $\Omega$  cannot be solved analytically in general. We solve for  $\Omega$  numerically using the QZ method. Once  $\Omega$  is solved,  $\Gamma$  and  $c$  follow straightforwardly.

The laws of motion of the state variables have been obtained endogenously, as functions of the parameters of the macroeconomic model. This contrasts with standard affine models, where both the equation for the short-term interest rate and the laws of motion of the state variables are postulated exogenously.

### 3 Macro-Finance Term Structure Model

The term structure of interest rates can be characterized by affine term structure models. These models are based on an explicit no-arbitrage condition in financial markets. The assumption of the absence of arbitrage opportunities seems quite natural for bond yields. Most bond markets are extremely liquid, and arbitrage opportunities are traded away immediately. Although a vast variety of affine term structure models exists due to the number of latent factors and the explicit formulation of their stochastic processes, they all share a common feature: in the single factor case the only risk factor equals the short rate, whereas in multi-factor cases the short rate is a combination of multiple risk factors. Monetary policy rules share the same structure, once the risk factors are interpreted as macroeconomic variables. Therefore, the short-term interest rate is a critical point of intersection between the finance and macroeconomic perspectives. From a finance perspective, the short rate is a fundamental building block for rates of other maturities because long yields are risk-adjusted averages of expected future short rates. From a macro perspective, the short rate is a key policy instrument under the direct control of the central bank, which adjusts it in order to achieve the economic stabilization goals of monetary policy. Together, the two perspectives suggest that understanding the manner in which central banks move the short rate (the policy rate) in response to macroeconomic shocks should explain movements in the short end of the yield curve. With the consistency between long and short rates enforced by the no-arbitrage assumption, macroeconomic shocks should account for movements in long-term yields as well. Combining the two lines of research could sharpen our understanding of the dynamics of the term structure of interest rates.

Dynamic term structure models have three basic components:

1. A collection of state variables. These state variables may be latent or observable such as macroeconomic variables.
2. A description of the dynamics of the state variables.
3. A mapping between the state variables and the term-structure of interest rates. The mapping can either be theoretically motivated and constructed so as to avoid arbitrage

opportunities or constructed solely based upon empirical considerations.

To build a term-structure model we require a number of assumptions. The first assumption is that the state vector influencing the term-structure of interest rates includes only macroeconomic variables. This means that the term-structure of interest rates is a function of a set of macroeconomic variables:

$$y_t^n = F(X_t, n) \quad (13)$$

where  $y_t^n$  is the yield to maturity of an  $n$ -period zero-coupon bond, and  $X_t$  is the vector of macroeconomic variables.

The second assumption is that there are no-arbitrage opportunities in the Mexican government bond market. The government bond market in Mexico is extremely liquid, so arbitrage opportunities would be traded away immediately by market participants. The assumption of no-arbitrage thus seems natural for Mexican bond yields. We use this assumption to develop the mapping from the state variables to the term structure of interest rates. First, we derive the relationship between the policy rate and the term structure of interest rates. Second, we relate the term structure to macroeconomic variables.

The no-arbitrage assumption is equivalent to the existence of a pricing kernel or stochastic discount factor that determines the values of all fixed-income securities. The pricing kernel is determined by investor's preferences for state-dependent payouts. Specifically, the value of an asset at time  $t$  equals  $E_t[M_{t+1}D_{t+1}]$ , where  $M_{t+1}$  is the pricing kernel, and  $D_{t+1}$  is the asset's value in  $t+1$  including any dividend or coupon paid by the asset. The pricing kernel process  $M_{t+1}$  prices all securities such that:

$$E_t[M_{t+1}R_{t+1}] = 1 \quad (14)$$

In particular, for an  $n$ -period bond,  $R_{t+1} = \frac{P_{t+1}^{n-1}}{P_t^n}$  where  $P_t^n$  denotes the time  $t$  price of an  $n$ -period zero-coupon bond. If  $M_{t+1} > 0$  for all  $t$ , the resulting returns satisfy the no-arbitrage condition (Harrison and Kreps 1979). Because we will be considering zero-coupon bonds, the payout from the bonds is simply their value in the following period, so that the

following recursive relationship holds:

$$P_t^n = E_t [M_{t+1} P_{t+1}^{n-1}] \quad (15)$$

The pricing kernel prices zero-coupon bonds from the no-arbitrage condition (15).  $P_t^n$  represents the price of an  $n$ -period zero-coupon bond, and the terminal value of the bond  $P_{t+n}^0$  is normalized to 1. To derive the term structure dynamics, we need to specify a process for the pricing kernel. Affine term structure models require linear state variable dynamics and an exponential affine pricing kernel process with conditionally normal shocks. For the state variable dynamics implied by the New-Keynesian model in equation (12) to fall in the affine class, we assume that the shocks are conditionally normally distributed with zero mean and variance-covariance matrix equal to  $\Sigma$ . Following the standard dynamic arbitrage-free term structure literature, we assume that the pricing kernel is conditionally log-normal, as follows:

$$M_{t+1} = \exp \left( -i_t - \frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \xi_{1,t+1} \right) \quad (16)$$

where  $\lambda_t$  are the time-varying market prices of risk associated with the source of uncertainty  $\xi_{1,t+1}$  in the economy. The market price of risk parameters are commonly assumed to be constant in Gaussian models or proportional to the factor volatilities. However, recent research (e.g. Dai and Singleton 2000), has highlighted the benefits in allowing for a more flexible specification of the market price of risk. We therefore assume that the market's required compensation for bearing risk can vary with the state of the economy. In particular, we assume that the prices of risk are affine in the state variables:

$$\lambda_t = \lambda_0 + \lambda_1 X_t \quad (17)$$

where  $X_t$  is defined by (12). The source of uncertainty in the small open economy pricing kernel is driven by the shocks to the macro variables. Equation (17) relates shocks in the underlying macroeconomic variables to the pricing kernel and therefore determines how shocks to macroeconomic variables affect the term-structure of interest rates. Note that in a micro-founded framework (Bekaert, Cho and Moreno 2005), the pricing kernel would

be linked to consumer preferences rather than being postulated exogenously. We prefer this exogenous specification because the pricing kernel postulated in equation (16) allows more flexibility in matching the behavior of the yield curve.

The constant risk premium parameter  $\lambda_0$  is a vector column, while the time varying risk premium parameter  $\lambda_1$  is a matrix. We assume that the time-varying risk premium parameter  $\lambda_1$  is a diagonal matrix. This reduces the number of parameters to be estimated.

The state dynamics (12), the pricing kernel (16), and the market prices of risk (17) form a discrete-time affine factor model. This model falls within the affine class of term structure models because bond prices are exponential affine functions of the state variables. More precisely, bond prices are given by:

$$P_t^n = \exp \left( \bar{A}_n + \bar{B}_n' X_t \right) \quad (18)$$

Using an induction argument and equations (12), (16), and (17), the coefficients  $\bar{A}_n$  and  $\bar{B}_n$  are derived from the cross-equation restrictions implied by the no-arbitrage condition (15). The cross-equation restrictions depend on parameters that describe the state dynamics and risk premia. The model is affine in the state vector, but the coefficients are nonlinear functions of the underlying parameters. In particular,  $\bar{A}_n$  and  $\bar{B}_n$  follow the difference equations:

$$\bar{A}_{n+1} = \bar{A}_1 + \bar{A}_n + \bar{B}_n' (c - \Gamma \Sigma \lambda_0) + \frac{1}{2} \bar{B}_n' \Gamma \Sigma \Gamma' \bar{B}_n \quad (19)$$

$$\bar{B}_{n+1}' = \bar{B}_n' (\Omega - \Gamma \Sigma \lambda_1) + \bar{B}_1' \quad (20)$$

Therefore, bond yields  $y_t^n$  are affine functions of the state variables:

$$y_t^n = -\frac{\log P_t^n}{n} = A_n + B_n' X_t \quad (21)$$

where  $A_n = -\frac{\bar{A}_n}{n}$ , and  $B_n = -\frac{\bar{B}_n}{n}$ .

The yield equation illustrates how the macroeconomic variables influence the term structure of interest rates. Each macroeconomic variable is a factor that describes the cross section of the term structure at a specific point in time. The zero-coupon yield curve is represented as an affine function of macroeconomic variables. The prices of risk control how long-term

yields respond relative to the short rate. The vector  $\lambda_0$  affects the long-run mean of yields because this vector affects the constant term in the yield equation, and the matrix  $\lambda_1$  affects the time-variation of risk premia, since it affects the slope coefficients in the yield equation.

Stacking all yields in a vector  $Y_t$ , we write the above equations jointly as:

$$Y_t = A_y + B_y X_t \quad (22)$$

## 4 Estimation Method

We estimate the model with monthly Mexican yields and Mexican and US macroeconomic data. The macroeconomic data are from July 2001 to June 2008.<sup>1</sup> The macroeconomic variables include core inflation, non-core inflation, the output gap, the nominal interest rate, the real exchange rate, the US inflation rate, the US output gap and the US nominal interest rate. The 1-month T-bill rates are used as the monetary policy instruments in both countries. The yield data are from July 2001 to June 2008, and include zero coupon yields of maturities 3, 6, 12, 24, 36, 60, 84 and 120 months.

Because of the estimation difficulty involved with a high dimension maximizing problem, we follow Ang and Piazzesi (2003) and estimate the model in two steps. In the first step, we use a GMM estimation technique to estimate the macro structural parameters with both US and Mexican data. Our estimation procedure finds parameter estimates that minimize the distance between the first and second moments from the model and those from the data.<sup>2</sup> In the second step, we fix these parameters, and estimate the risk premium parameters of the term structure model by maximum likelihood with Mexican yield data, and with Mexican and US macroeconomic data. This estimation technique helps to ensure that the macro parameters are not distorted by the estimation algorithm in an effort to fit the zero-coupon cross section.

This model provides a particular convenient form for the joint dynamics of the macro variables and the term structure of interest rates.

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<sup>1</sup>Chiquiar, Noriega and Ramos Francia (2007) find that inflation in Mexico seems to have switched from a non-stationary process to a stationary process around the end of 2000 or the beginning of year 2001.

<sup>2</sup>Sidaoui and Ramos-Francia (2008) estimate with GMM the Euler equations that characterize the equilibrium conditions of small-scale macro model for Mexico using different samples.

Let  $Z_t = [X_t', Y_t']'$ , where  $Y_t = (y_t^3, y_t^6, y_t^{12}, y_t^{24}, y_t^{36}, y_t^{60}, y_t^{84}, y_t^{120})'$ .

Consequently the model that needs to be estimated is the following:

$$X_t = c + \Omega X_{t-1} + \Gamma \xi_t \quad (23)$$

$$Z_t = A_Z + B_Z X_t \quad (24)$$

$$\text{where } A_Z = \begin{bmatrix} 0_{n_1 \times 1} \\ A_y \end{bmatrix}, B_Z = \begin{bmatrix} I_{n_1 \times n_1} \\ B_y \end{bmatrix}$$

where  $n_1$  is the number of state variables and

$$A_y = \begin{bmatrix} A_3 \\ A_6 \\ A_{12} \\ A_{24} \\ A_{36} \\ A_{60} \\ A_{84} \\ A_{120} \end{bmatrix}, B_y = \begin{bmatrix} B'_3 \\ B'_6 \\ B'_{12} \\ B'_{24} \\ B'_{36} \\ B'_{60} \\ B'_{84} \\ B'_{120} \end{bmatrix}$$

## 4.1 MLE Estimation

We now describe the general method we use to estimate the processes governing the risk premium parameters in  $\lambda_t$  with the data described above.

### 4.1.1 State-Space form

For a given set of observable variables, the likelihood function of this model can be calculated, and the model can be estimated by maximum likelihood. The yields themselves are analytical functions of the state variables  $X_t$ . We use the common approach in the finance literature of assuming that yields are measured with error to prevent stochastic singularity. In addition,

we assume that measurement error shocks and shocks to the state variables are orthogonal. Using  $\tilde{X}_t = [X'_t, 1]'$ , we find:

$$\tilde{X}_{t+1} = A\tilde{X}_t + B\xi_{t+1} \quad (25)$$

$$Z_t = C\tilde{X}_t + w_t \quad (26)$$

$$w_t = Dw_{t-1} + \eta_t \quad (27)$$

where

$$A = \begin{bmatrix} \Omega & c \\ 0_{1 \times n_1} & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} \Gamma \\ 0_{1 \times n_1} \end{bmatrix}$$

$$C = \begin{bmatrix} B_z & A_z \end{bmatrix}$$

$w$  represents measurement error and elements of  $D$  are the parameters governing serial correlation of the measurement error. We assume that  $E_t \eta_t \eta'_t = R$ , and  $E_t \epsilon_t \eta'_s = 0$  for all periods  $t$  and  $s$ . Define the quasi-differenced process  $\bar{Z}_t$  as:

$$\bar{Z}_t = Z_{t+1} - DZ_t \quad (28)$$

Then we can rewrite the system as:

$$\tilde{X}_{t+1} = A\tilde{X}_t + B\xi_{t+1} \quad (29)$$

$$\bar{Z}_t = \bar{C}\tilde{X}_t + CB\xi_{t+1} + \eta_{t+1} \quad (30)$$

where  $\bar{C} = CA - DC$ .



### 4.1.2 Log-likelihood function

$$\ln L(\Theta) = \sum_{t=0}^{T-1} \{ \ln \det(\Omega_t) + \text{trace}(\Omega_t^{-1} u_t u_t') \} \quad (31)$$

The parameters to be estimated are stacked in the vector  $\Theta$ , the innovation vector is  $u_t$ , and its covariance matrix is  $\Omega_t$ .

The innovation vector  $u_t$  and its covariance  $\Omega_t$  are defined as follows:

$$\begin{aligned} u_t &= \bar{Z}_t - E \left[ \bar{Z}_t \mid \bar{Z}_{t-1}, \bar{Z}_{t-2}, \dots, \bar{Z}_0, \hat{X}_0 \right] \\ &= Z_{t+1} - E \left[ Z_{t+1} \mid Z_t, Z_{t-1}, \dots, Z_0, \hat{X}_0 \right] \\ &= Z_{t+1} - DZ_t - \bar{C}\hat{X}_t \end{aligned}$$

which depends on the predicted state  $\hat{X}_t$ :

$$\hat{X}_t = E \left[ \tilde{X}_t \mid Z_t, Z_{t-1}, \dots, Z_0, \hat{X}_0 \right]$$

$$\Omega_t = E u_t u_t' = \bar{C} \Sigma_t \bar{C}' + R + C B B' C'$$

The predicted state evolves according to:

$$\hat{X}_{t+1} = A \hat{X}_t + K_t u_t$$

where  $K_t$ , and  $\Sigma_t$  are the Kalman gain and state covariance associated with the Kalman filter respectively.

$$\begin{aligned} K_t &= (B B' C' + A \Sigma_t \bar{C}') \Omega_t^{-1} \\ \Sigma_{t+1} &= A \Sigma_t A' + B B' - (B B' C' + A \Sigma_t \bar{C}') \Omega_t^{-1} (\bar{C} \Sigma_t A' + C B B') \end{aligned}$$

An innovations representations for the system is:

$$\hat{X}_{t+1} = A \hat{X}_t + K_t u_t \quad (32)$$

$$u_t = \bar{Z}_t - \bar{C} \hat{X}_t \quad (33)$$

For the maximum likelihood estimation, we fix the macro structural parameters and

estimate the term-structure parameters.

## 5 Results

Section 5.1 interprets the parameter estimates of the macro-finance term structure model. To determine the effect of the addition of macro factors into term structure models, we look at impulse response functions of macro variables and yields to the underlying macro shocks in section 5.2.

### 5.1 Parameter estimates

The macroeconomic structural parameters are broadly in line with existing evidence based on Mexican (monthly) data so we will not analyze them here.<sup>3</sup> We concentrate instead on the term-structure parameters. Tables 1 and 2 present the market price of risk parameter estimates and their standard errors. The dynamics of the term-structure of interest rates depend on the short-term interest rate, and on the risk premia parameters  $\lambda_0$  and  $\lambda_1$ . A non-zero vector  $\lambda_0$  affects the long-run mean of yields because this parameter affects the constant term in the yield equation (21). Table 1 presents the constant risk premia parameter estimates  $\lambda_0$  with standard errors in parentheses. The data generating and the risk neutral measures coincide if  $\lambda_t = 0$  for all  $t$ . This case is called the "Expectations Hypothesis". Macro models typically use the Expectations Hypothesis to infer long term yield dynamics from short rates. In the Vasicek (1977) model,  $\lambda_0$  is non-zero and  $\lambda_1$  is zero, which allows the average yield curve to be upward sloping, but does not allow risk premia to be time-varying. Negative parameters in the estimated vector  $\lambda_0$  induce the unconditional mean of the short rate under the risk-neutral measure to be higher than under the data-generating measure. Given that bond prices are computed under the risk-neutral measure, negative parameters in  $\lambda_0$  induce long yields to be on average higher than short yields and the average yield curve to be upward sloping.

Time-variation in risk premia is driven by the parameters in  $\lambda_1$ . These parameters affect the time-variation of risk-premia, since they affect the slope coefficients in the yield equation

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<sup>3</sup>These parameter estimates are presented in Appendix 2.

Table 1  
Parameter estimates with standard errors

Parameter	Estimate	Std. Error
$\lambda_{0,\pi^c}$	-0.16	(0.003)
$\lambda_{0,i}$	-1.10	(0.12)
$\lambda_{0,q}$	0.23	(0.003)
$\lambda_{0,x}$	0.94	(0.45)
$\lambda_{0,i^{USA}}$	1.45	(0.48)
$\lambda_{0,\pi^{USA}}$	-2.23	(0.84)
$\lambda_{0,\pi^{nc}}$	-0.16	(0.003)
$\lambda_{0,x^{USA}}$	0.06	(0.004)

Table 2  
Parameter estimates with standard errors

Parameter	Estimate	Std. Error
$\lambda_{1,\pi^c}$	-0.04	(0.008)
$\lambda_{1,i}$	-0.25	(0.01)
$\lambda_{1,q}$	0.03	(0.001)
$\lambda_{1,x}$	0.08	(0.005)
$\lambda_{1,i^{USA}}$	1.24	(0.003)
$\lambda_{1,\pi^{USA}}$	-0.36	(0.13)
$\lambda_{1,\pi^{nc}}$	-0.04	(0.008)
$\lambda_{1,x^{USA}}$	1.92	(0.001)

(21). The more negative the terms on  $\lambda_1$ , the more positively long-term yields react to positive factor shocks. Table 2 reports the time-varying risk premium parameters with the restriction that the matrix parameter  $\lambda_1$  is diagonal. Table 2 shows that all the diagonal elements of  $\lambda_1$  are statistically significant. The parameter estimates indicate that risk premia vary significantly over time. As in developed markets, results from the estimation of the model show that term premia are countercyclical, and that they increase with the level of the inflation rate. The parameter  $\lambda_{1x}$  is positive. This means that positive demand shocks decrease term premiums. Booms tend to make investors more willing to hold long term bonds, while they require a larger premium during recessions. The parameter  $\lambda_{1\pi^c}$  is negative. This means that the inflation premium is increasing in the level of the inflation rate. Higher inflation makes long-term bonds riskier and increases the premium that investors require to hold them.

## 5.2 Impulse response functions

Our structural model allows us to compute impulse response functions of macro variables and yields to the underlying macro shocks. In this section we characterize the dynamics implied by the term-structure model using standard impulse response functions. The following figures show the impulse responses to monetary policy shocks, cost-push shocks and demand shocks.

We show the responses of the macroeconomic variables as well as the responses of yields to the underlying macro shocks. We start from Figure 1, which displays the impulse responses to a monetary policy shock. This shock reflects shifts to the short-term interest rate unexplained by neither the output gap nor the inflation gap. A contractionary monetary policy shock yields a strong response of both cyclical output and inflation. The interest rate increases following the monetary policy shock, but after some periods it undershoots its steady-state level. This undershooting is related to the endogenous decrease of cyclical output and inflation to the monetary policy shock. The response of the yield curve is decreasing in the maturity of yields. As expected, the initial shock of a 1% increase in the short rate dies out gradually across the yield curve. Hence, a monetary policy shock tends to cause a flattening of the yield curve. The term spreads narrow from an unexpected monetary policy shock. Figure 2 plots the contemporaneous response of the yield curve to a monetary policy shock. This shock raises all yields on impact but the initial response is highest for the short yield, while the initial response of the medium and long yields is small. Hence, the slope of the yield curve decreases after a monetary policy shock on impact.

Figure 3 shows the impulse responses to a cost-push shock. The monetary authority increases the short-term interest rate following a cost-push shock. The interest rate moves slowly because of the high estimated interest rate smoothing coefficient in the policy rule. The real interest rate decreases initially, but then it increases above its steady-state level for several periods. Output increases initially, but then it exhibits a hump-shaped decline for several periods. A cost-push shock raises the level of all yields. The rise in yields is highest at medium-term maturities around the 2-year maturity. The very short ones move slowly because of the interest rate smoothing coefficient in the policy rule. Cost-push shocks cause

a very persistent steepening upward shift in the yield curve.

One advantage of our joint treatment of macroeconomics and term-structure dynamics is that we are able to analyze the behavior of risk premia. In our model the risk premium varies over time and increases or decreases as a function of the state variables. The risk premium on nominal bonds is an affine function of the state variables. At times when inflation is procyclical as will be the case if the macroeconomy moves along a stable Phillips Curve, nominal bonds are countercyclical, making nominal bonds desirable hedges against business cycle risk. At times when inflation is countercyclical, as will be the case if the economy is affected by a cost-push shock that shifts the Phillips Curve, nominal bond returns are procyclical. In this context, investors demand a positive risk premium to hold assets whose payoffs are procyclical. Figure 4 shows the impulse responses of the yields to the cost-push shock for the case in which risk premia are time-varying (TVRP), and for the case in which risk premia are constant (CRP). Risk premia increase after a cost-push shock, implying that yields increase more when risk-premia are time-varying. Such increase in the yield premium is highly significant from an economic viewpoint, as it plays a large quantitative role in shaping the total yield responses displayed in the time-varying risk premia case. Figure 5 plots the contemporaneous response of the yield curve to a cost-push shock for the case in which risk premia are time-varying (TVRP), and for the case in which risk premia are constant (CRP). The yield curve increases more in the time-varying risk premia case because risk premia increases after a positive cost-push shock. Higher inflation makes long-term bond riskier and increases the premium that investors require to hold them.

Figure 6 shows the impulse responses to a demand shock, which can also be interpreted as a preference shock. The demand shock increases output and inflation, so the monetary authority increases the short-term interest rate following this shock. Demand shocks also increase all yields, but the effect is smaller for long-term yields. A demand shock causes a flattening upward shift in the yield curve. Due to the policy response, the yield curve increases more at the short and medium term maturities, and moves little at the long end. Hence, the term spreads narrow from an unexpected demand shock.

Figure 7 shows contemporaneous response of the yield curve to a demand shock. Positive demand shocks lead to an upward flattening shift in the yield curve. The flattening of the

curve is explained by the monetary policy response and its effect on inflationary expectations, and by the time-varying term premia.

## 6 Conclusions

We have developed and estimated a model that combines an affine no-arbitrage finance specification of the term structure with a macroeconomic model of a small open economy to analyze how different macroeconomic shocks affect the term-structure of interest rates in Mexico. Our key findings are as follows. Term-premia in the Mexican government bond market are time-varying. Results from the model show that term premia are countercyclical, and that they increase with the level of the inflation rate. In addition, our model delivers strong contemporaneous responses of the entire term structure to various macroeconomic shocks. For example, shocks that are perceived to have a persistent effect on inflation affect the level of the yield curve. The effect on medium and long-term yields results from the increase in expected future short rates and in risk premia. With respect to demand shocks, our results show that a positive demand shock leads to an upward flattening shift in the yield curve. The flattening of the curve is explained by both the monetary policy response and the time-varying term premia.

We are not aware of any model that combines the finance and macroeconomic perspectives of the term structure of interest rates for small open emerging economies. Our results show that combining these two lines of research helps in understanding the macroeconomic determinants of the term structure of interest rates. The no-arbitrage framework provides a complete description of how the yields of all maturities respond to the shocks to the underlying state variables, and the macro model introduces structure on the dynamics of the macro variables and thus allows us to identify how structural shocks affect the economy.

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## 8 Appendix 1

Figure 1

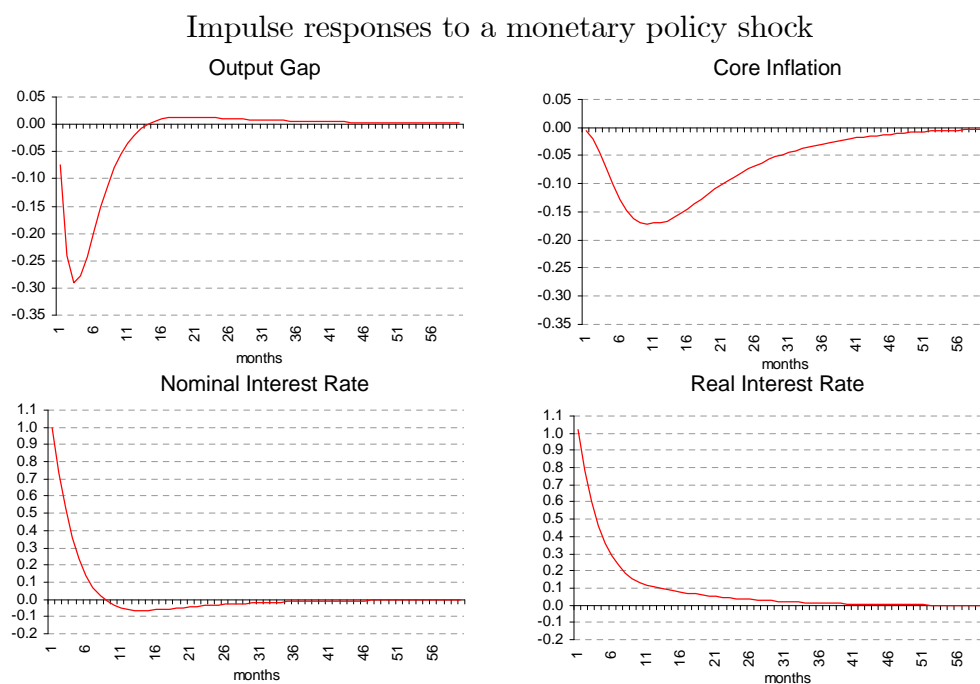


Figure 1 (cont)

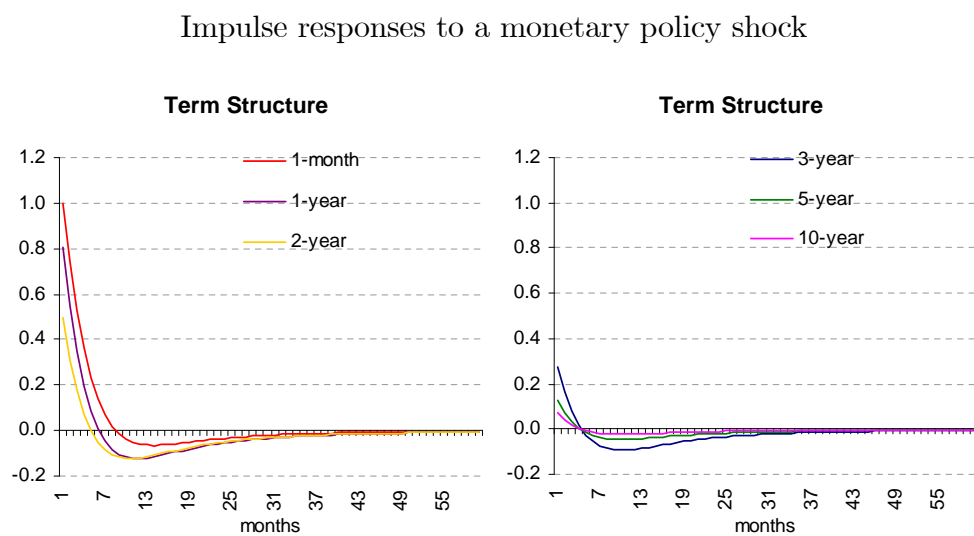


Figure 2

Contemporaneous response of the yield-curve to a monetary policy

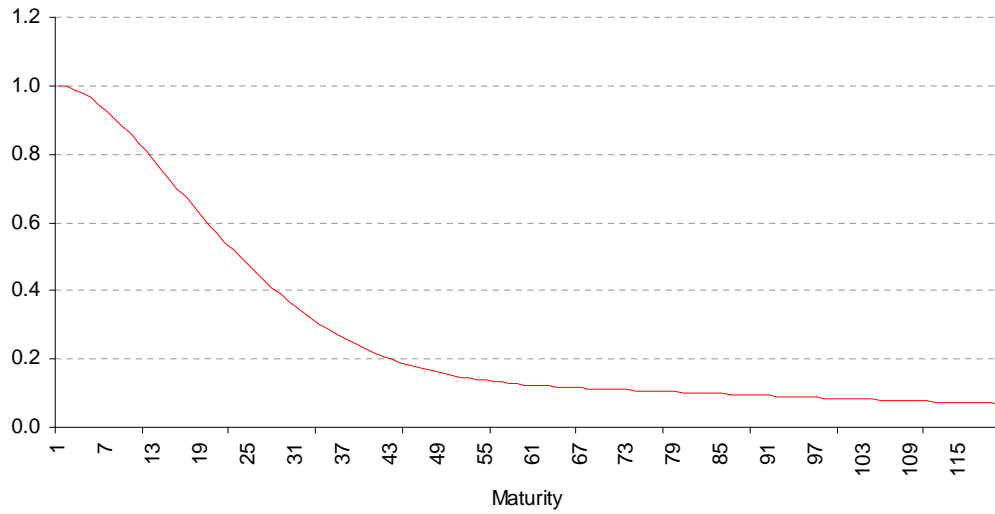


Figure 3

Impulse responses to a cost-push shock

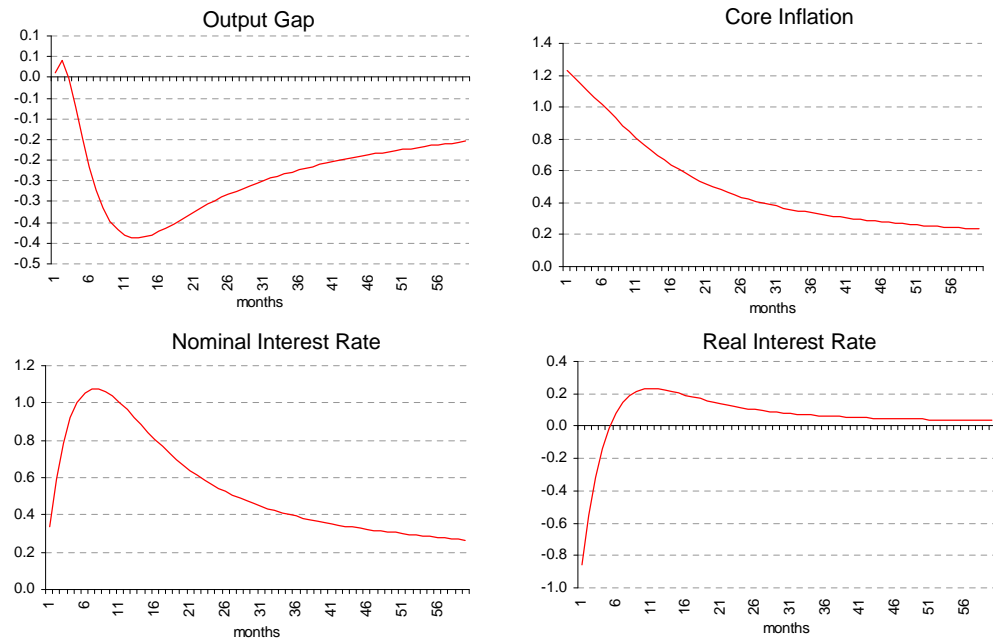


Figure 3 (cont.)

### Impulse responses to a cost-push shock

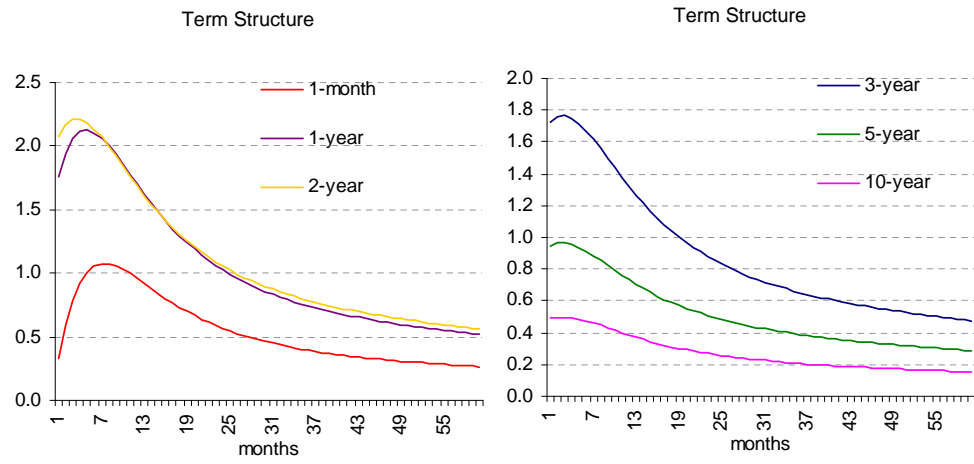


Figure 4

### Impulse responses to a cost-push shock. Time-varying and constant risk

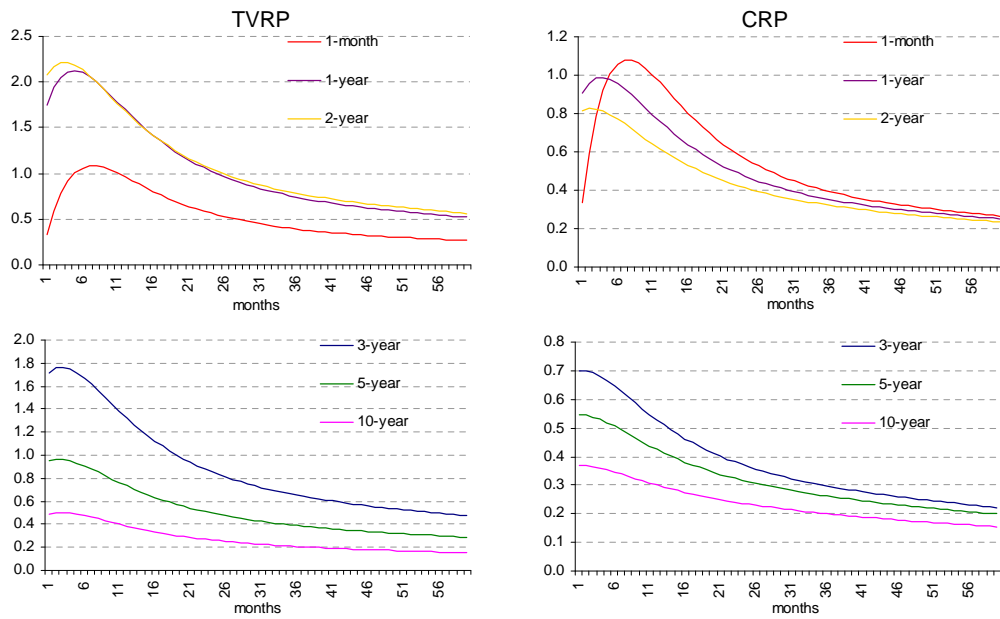


Figure 5  
Contemporaneous response of the yield-curve to a cost-push

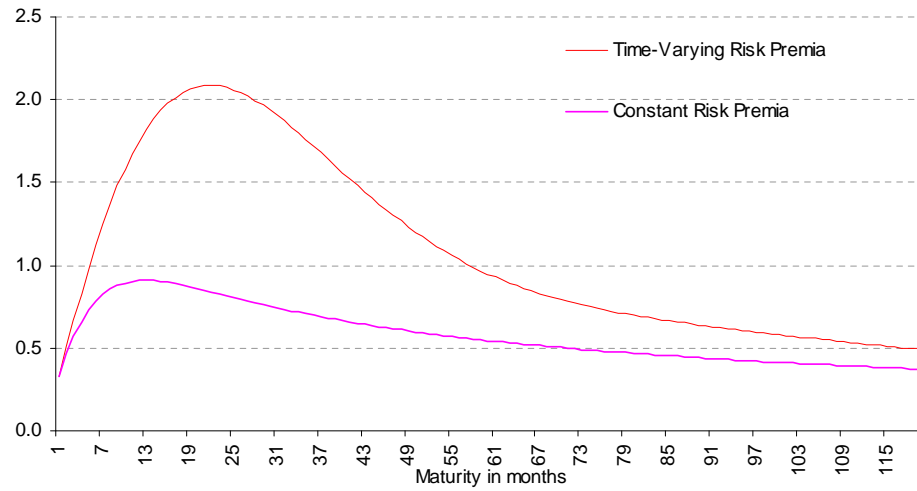


Figure 6  
Impulse responses to a demand shock

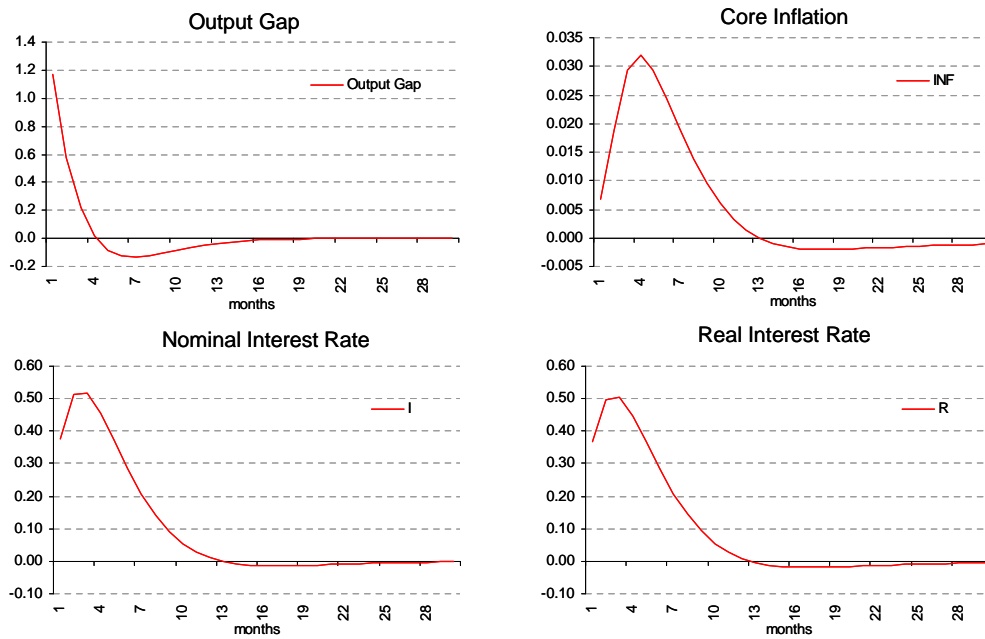


Figure 6 (cont.)

Impulse responses to a demand shock

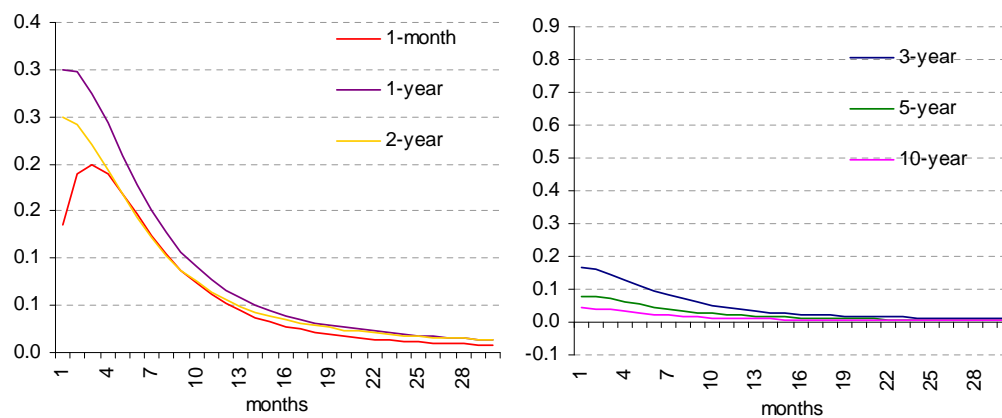
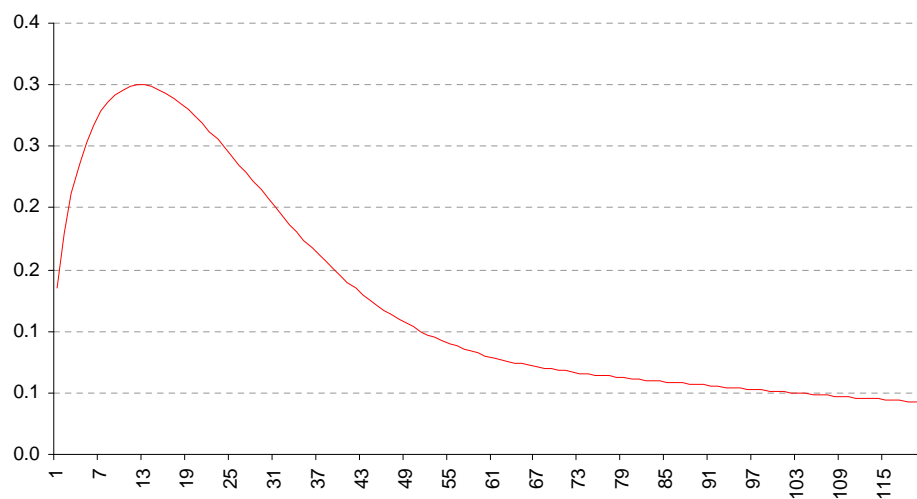


Figure 7

Contemporaneous response of the yield curve to a demand shock



## 9 Appendix 2

This appendix describes the estimation of the macroeconomic parameters. The equations that characterize the equilibrium of the small open economy are the following:

(i)	Phillips Curve	$\pi_t^c = a_1 \pi_{t-1}^c + a_2 E_t [\pi_{t+1}^c] + a_3 x_t + a_4 (\Delta e_t + \pi_t^{USA}) + \varepsilon_t^{AS}$
(ii)	IS Curve	$x_t = b_1 x_{t-1} + b_2 E_t [x_{t+1}] + b_3 (i_t - E_t \pi_{t+1}) + b_4 x_t^{USA} + b_5 q_t + \varepsilon_t^{IS}$
(iii)	Real Exchange Rate	$q_t = c_1 (q_{t-1}) + c_2 (E_t [q_{t+1}]) + (i_t^{USA} - E_t \pi_t^{USA}) - (i_t - E_t \pi_t) + \varepsilon_t^q$
(iv)	Taylor Rule	$i_t = (1 - \rho)(d_1 (\pi_t - \pi_t^*) + d_2 x_t) + \rho i_{t-1} + \varepsilon_t^{MP}$
(v)	Inflation	$\pi_t \equiv \omega \pi_t^c + (1 - \omega) \pi_t^{nc}$

$\pi_t$  denotes the headline inflation rate,  $\pi_t^c$  the core inflation rate,  $\pi_t^{nc}$  the non-core inflation rate,  $\pi_t^*$  is the inflation target,  $x_t$  the output gap,  $q_t$  the real exchange rate,  $e_t$  the nominal exchange rate,  $i_t$  the nominal interest rate, and  $i_t^{USA}$ ,  $\pi_t^{USA}$  and  $x_t^{USA}$  denote respectively the US nominal interest rate, US monthly inflation and the US output gap. The headline inflation rate is defined using the weights of the core and non-core price sub-indices on the CPI which implies that  $\omega$  is equal to 0.69.

The following tables present the estimated parameters.

Table 3  
Phillips Curve

	<b>a<sub>1</sub></b>	<b>a<sub>2</sub></b>	<b>a<sub>3</sub></b>	<b>a<sub>4</sub></b>
Coefficient	0.46	0.52	0.04	0.02
Std. Error	(0.0007)	(0.0008)	(0.0000)	(0.0002)

Table 4  
IS equation

	<b>b<sub>1</sub></b>	<b>b<sub>2</sub></b>	<b>b<sub>3</sub></b>	<b>b<sub>4</sub></b>	<b>b<sub>5</sub></b>
Coefficient	0.47	0.30	-0.1	0.16	1.03
Std. Error	(0.067)	(0.07)	(0.02)	(0.02)	(0.2)

Table 5

RER equation

	$\mathbf{c}_1$	$\mathbf{c}_2$
Coefficient	0.53	0.47
Std. Error	(0.0049)	(0.003)

Table 6

Taylor rule

	$\mathbf{d}_1$	$\mathbf{d}_2$	$\boldsymbol{\rho}$
Coefficient	1.32	2.48	0.87
Std. Error	(0.25)	(0.28)	(0.07)